

1.2 | Functions and Graphs

You will be able to use the language, notation, and graphical representation of functions to express relationships between variable quantities.

- Function, domain, and range
- Interval notation
- Graphs in the coordinate plane
- Odd and even symmetry
- Piecewise-defined functions, including absolute value
- Composite functions

Functions

We assumed some familiarity with the language of functions and graphs in Section 1.1, where we connected lines in the coordinate plane with their algebraic representations, called linear functions. In this section we will review some of the other terminology associated with functions and graphs so that you will be comfortable with them in the chapters to come.

DEFINITION Function (and Related Terms)

A **function** from a set D to a set R is a rule that assigns to every element in D a unique element in R . The set of all input values in D is the **domain** of the function, and the set of all output values in R is the **range** of the function. In **function notation**, we use $f(x)$ to denote the range value that f assigns to the domain value x . The set of all points (x, y) in the coordinate plane determined by the rule $y = f(x)$ is the **graph of the function f** . The variable x is the **independent variable** and the variable y is the **dependent variable**.

Most of the functions in this course will be from the real numbers to the real numbers. Indeed, this course might be more fully described as “the calculus of real functions of a real variable.” In this context, the domain is a subset (not always the whole set) of the real numbers, as is the range.

The uniqueness in the definition of $f(x)$ is essential. It might be helpful to think of f as a machine that assigns a unique range value to every domain value (Figure 1.6). If the same x were put into the same machine, the same $f(x)$ would result. On the other hand, two (or more) different domain values could be assigned the same range value (Figure 1.7).



Figure 1.6 A “machine” diagram for a function.

Leonhard Euler (1707–1783)



Leonhard Euler, the dominant mathematical figure of his century and the most prolific mathematician ever, was also an astronomer, physicist, botanist, and chemist, and an expert in oriental languages. His work was the first to give the function concept the prominence that it has in mathematics today. Euler's collected books and papers fill 72 volumes. This does not count his enormous correspondence to approximately 300 addresses. His introductory algebra text, written originally in German (Euler was Swiss), is still available in English translation.

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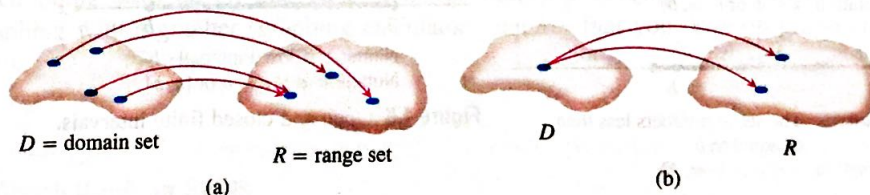


Figure 1.7 (a) A function from a set D to a set R . (b) Not a function. The assignment is not unique.

Euler invented a symbolic way to say “ y is a function of x ”:

$$y = f(x),$$

which we read as “ y equals f of x .” This notation enables us to give different functions different names by changing the letters we use. To say that the boiling point of water is a function of elevation, we can write $b = f(e)$. To say that the area of a circle is a function of the circle's radius, we can write $A = A(r)$, giving the function the same name as the dependent variable.

The notation $y = f(x)$ gives a way to denote specific values of a function. The value of f at a can be written as $f(a)$, read “ f of a .”

EXAMPLE 1 The Circle-Area Function

Write a formula that expresses the area of a circle as a function of its radius. Use the formula to find the area of a circle of radius 2 in.

SOLUTION

If the radius of the circle is r , then the area $A(r)$ of the circle can be expressed as $A(r) = \pi r^2$. The area of a circle of radius 2 can be found by evaluating the function $A(r)$ at $r = 2$.

$$A(2) = \pi(2)^2 = 4\pi$$

The area of a circle of radius 2 in is 4π in².

Now Try Exercise 3.

Domains and Ranges

When we define a function $y = f(x)$ with a formula and the domain is not stated or restricted by context, the domain is assumed to be the largest set of x values for which the formula gives real y values. This is the **implied domain**. In Example 1, the domain of the function is restricted by context, since the independent variable is a radius and must be nonnegative. The restricted domain is called the **relevant domain**. The domain of a function can also be restricted as part of its definition, as in the function $v(t) = 32t + 25$ for $t \geq 0$.

The domains and ranges of many real-valued functions of a real variable are intervals or combinations of intervals. The intervals may be open, closed, or half-open (Figures 1.8 and 1.9) and finite or infinite (Figure 1.10).

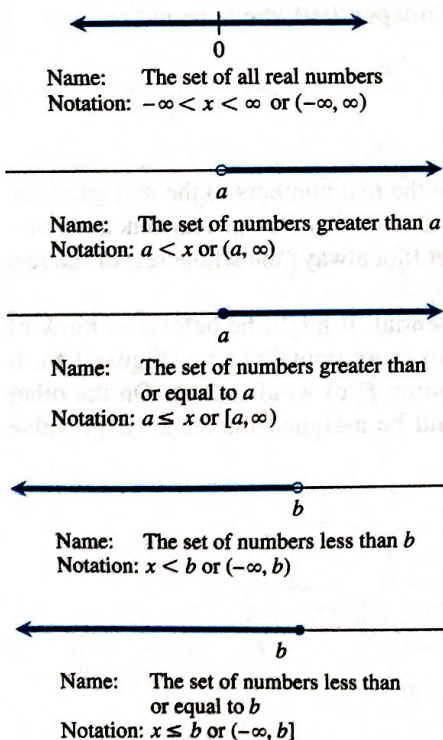


Figure 1.10 Infinite intervals—rays on the number line and the number line itself. The symbol ∞ (infinity) is used merely for convenience; it does not mean there is a number ∞ .

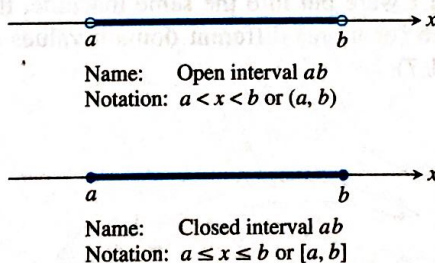


Figure 1.8 Open and closed finite intervals.

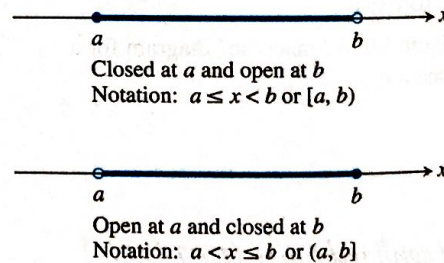


Figure 1.9 Half-open finite intervals.

The endpoints of an interval make up the interval’s **boundary** and are called **boundary points**. The remaining points make up the interval’s **interior** and are called **interior points**. **Closed intervals** contain their boundary points. **Open intervals** contain no boundary points. Every point of an open interval is an interior point of the interval.

Viewing and Interpreting Graphs

The points (x, y) in the plane whose coordinates are the input-output pairs of a function $y = f(x)$ make up the function’s graph. The graph of the function $y = x + 2$, for example, is the set of points with coordinates (x, y) for which y equals $x + 2$.

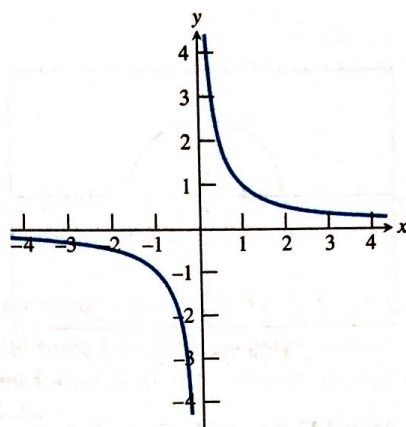
EXAMPLE 2 Identifying Domain and Range of a Function

Identify the domain and range, and then sketch a graph of the function.

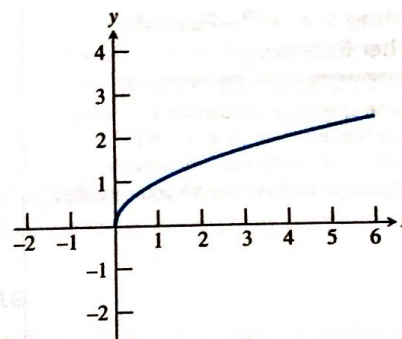
(a) $y = \frac{1}{x}$ (b) $y = \sqrt{x}$

SOLUTION

(a) The formula gives a real y value for every real x value except $x = 0$. (We cannot divide any number by 0.) The domain is $(-\infty, 0) \cup (0, \infty)$. The value y takes on every real number except $y = 0$. ($y = c \neq 0$ if $x = 1/c$.) The range is also $(-\infty, 0) \cup (0, \infty)$. A sketch is shown in Figure 1.11a.



(a)



(b)

Figure 1.11 A sketch of the graph of (a) $y = 1/x$ and (b) $y = \sqrt{x}$. (Example 2)

(b) The formula gives a real number only when x is positive or zero. The domain is $[0, \infty)$. Because \sqrt{x} denotes the principal square root of x , y is greater than or equal to zero. The range is also $[0, \infty)$. A sketch is shown in Figure 1.11b.

Now Try Exercise 9.

Graphing with pencil and paper requires that you develop graph *drawing* skills. Graphing with a grapher (graphing calculator) requires that you develop graph *viewing* skills.

Power Function

Any function that can be written in the form $f(x) = kx^a$, where k and a are nonzero constants, is a **power function**.

Graph Viewing Skills

1. Recognize that the graph is reasonable.
2. See all the important characteristics of the graph.
3. Interpret those characteristics.
4. Recognize grapher failure.

Being able to recognize that a graph is reasonable comes with experience. You need to know the basic functions, their graphs, and how changes in their equations affect the graphs.

Grapher failure occurs when the graph produced by a grapher is less than precise—or even incorrect—usually due to the limitations of the screen resolution of the grapher.

EXAMPLE 3 Identifying Domain and Range of a Function

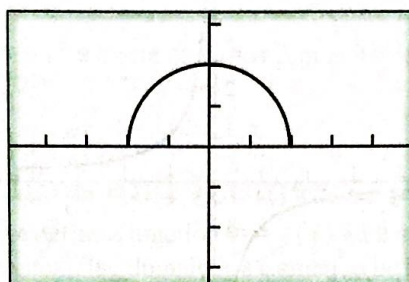
Use a grapher to identify the domain and range, and then draw a graph of the function.

(a) $y = \sqrt{4 - x^2}$ (b) $y = x^{2/3}$

SOLUTION

(a) Figure 1.12a shows a graph of the function for $-4.7 \leq x \leq 4.7$ and $-3.1 \leq y \leq 3.1$, that is, the viewing window $[-4.7, 4.7]$ by $[-3.1, 3.1]$, with x -scale = y -scale = 1. The graph appears to be the upper half of a circle. The domain appears to be $[-2, 2]$. This observation is correct because we must have $4 - x^2 \geq 0$, or equivalently, $-2 \leq x \leq 2$. The range appears to be $[0, 2]$, which can also be verified algebraically.

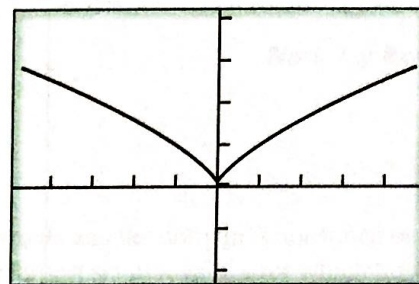
$y = \sqrt{4 - x^2}$



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

(a)

$y = x^{2/3}$



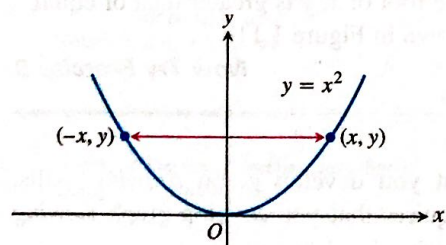
$[-4.7, 4.7]$ by $[-2.1, 4.1]$

(b)

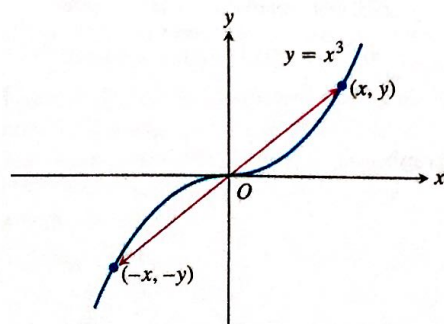
Figure 1.12 The graph of (a) $y = \sqrt{4 - x^2}$ and (b) $y = x^{2/3}$. (Example 3)

(b) Figure 1.12b shows a graph of the function in the viewing window $[-4.7, 4.7]$ by $[-2.1, 4.1]$, with x -scale = y -scale = 1. The domain appears to be $(-\infty, \infty)$, which we can verify by observing that $x^{2/3} = (\sqrt[3]{x})^2$. Also the range is $[0, \infty)$ by the same observation.

Now Try Exercise 15.



(a)



(b)

Figure 1.13 (a) The graph of $y = x^2$ (an even function) is symmetric about the y -axis. (b) The graph of $y = x^3$ (an odd function) is symmetric about the origin.

Even Functions and Odd Functions—Symmetry

The graphs of *even* and *odd* functions have important symmetry properties.

DEFINITIONS Even Function, Odd Function

A function $y = f(x)$ is an

even function of x if $f(-x) = f(x)$,

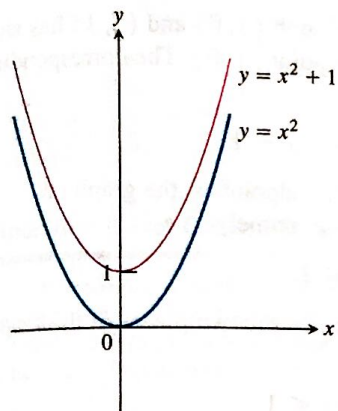
odd function of x if $f(-x) = -f(x)$,

for every x in the function's domain.

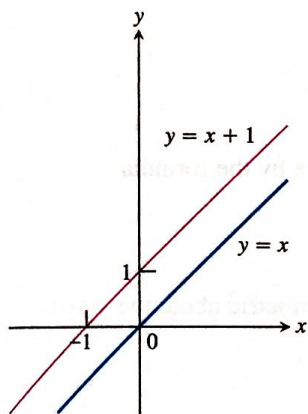
The names even and odd come from powers of x . If y is an even power of x , as in $y = x^2$ or $y = x^4$, it is an even function of x (because $(-x)^2 = x^2$ and $(-x)^4 = x^4$). If y is an odd power of x , as in $y = x$ or $y = x^3$, it is an odd function of x (because $(-x)^1 = -x$ and $(-x)^3 = -x^3$).

The graph of an even function is **symmetric about the y -axis**. Since $f(-x) = f(x)$, a point (x, y) lies on the graph if and only if the point $(-x, y)$ lies on the graph (Figure 1.13a).

The graph of an odd function is **symmetric about the origin**. Since $f(-x) = -f(x)$, a point (x, y) lies on the graph if and only if the point $(-x, -y)$ lies on the graph (Figure 1.13b).



(a)



(b)

Figure 1.14 (a) When we add the constant term 1 to the function $y = x^2$, the resulting function $y = x^2 + 1$ is still even and its graph is still symmetric about the y -axis. (b) When we add the constant term 1 to the function $y = x$, the resulting function $y = x + 1$ is no longer odd. The symmetry about the origin is lost. (Example 4)

Equivalently, a graph is symmetric about the origin if a rotation of 180° about the origin leaves the graph unchanged.

EXAMPLE 4 Recognizing Even and Odd Functions

- $f(x) = x^2$ Even function: $(-x)^2 = x^2$ for all x ; symmetry about y -axis.
- $f(x) = x^2 + 1$ Even function: $(-x)^2 + 1 = x^2 + 1$ for all x ; symmetry about y -axis (Figure 1.14a).
- $f(x) = x$ Odd function: $(-x) = -x$ for all x ; symmetry about the origin.
- $f(x) = x + 1$ Not odd: $f(-x) = -x + 1$, but $-f(x) = -x - 1$. The two are not equal.
Not even: $(-x) + 1 \neq x + 1$ for all $x \neq 0$ (Figure 1.14b).

Now Try Exercises 21 and 23.

It is useful in graphing to recognize even and odd functions. Once we know the graph of either type of function on one side of the y -axis, we know its graph on both sides.

Piecewise-Defined Functions

While some functions are defined by single formulas, others are defined by applying different formulas to different parts of their domains.

EXAMPLE 5 Graphing Piecewise-Defined Functions

$$\text{Graph } y = f(x) = \begin{cases} -x, & x < 0 \\ x^2, & 0 \leq x \leq 1 \\ 1, & x > 1. \end{cases}$$

SOLUTION

The values of f are given by three separate formulas: $y = -x$ when $x < 0$, $y = x^2$ when $0 \leq x \leq 1$, and $y = 1$ when $x > 1$. However, the function is *just one function*, whose domain is the entire set of real numbers (Figure 1.15).

Now Try Exercise 33.

EXAMPLE 6 Writing Formulas for Piecewise Functions

Write a formula for the function $y = f(x)$ whose graph consists of the two line segments in Figure 1.16.

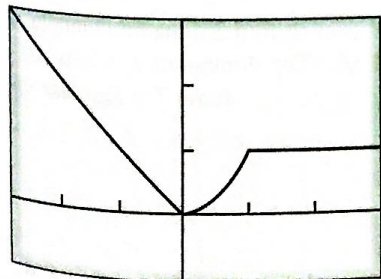
SOLUTION

We find formulas for the segments from $(0, 0)$ to $(1, 1)$ and from $(1, 0)$ to $(2, 1)$ and piece them together in the manner of Example 5.

Segment from $(0, 0)$ to $(1, 1)$ The line through $(0, 0)$ and $(1, 1)$ has slope $m = (1 - 0)/(1 - 0) = 1$ and y -intercept $b = 0$. Its slope-intercept equation is $y = x$. The segment from $(0, 0)$ to $(1, 1)$ that includes the point $(0, 0)$ but not the point $(1, 1)$ is the graph of the function $y = x$ restricted to the half-open interval $0 \leq x < 1$, namely,

$$y = x, \quad 0 \leq x < 1.$$

$$y = \begin{cases} -x, & x < 0 \\ x^2, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$



$[-3, 3]$ by $[-1, 3]$

Figure 1.15 The graph of a piecewise-defined function. (Example 5)

continued

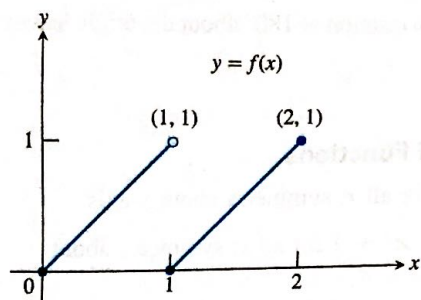


Figure 1.16 The segment on the left contains $(0, 0)$ but not $(1, 1)$. The segment on the right contains both of its endpoints. (Example 6)

Segment from $(1, 0)$ to $(2, 1)$ The line through $(1, 0)$ and $(2, 1)$ has slope $m = (1 - 0)/(2 - 1) = 1$ and passes through the point $(1, 0)$. The corresponding point-slope equation for the line is

$$y = 1(x - 1) + 0, \text{ or } y = x - 1.$$

The segment from $(1, 0)$ to $(2, 1)$ that includes both endpoints is the graph of $y = x - 1$ restricted to the closed interval $1 \leq x \leq 2$, namely,

$$y = x - 1, \quad 1 \leq x \leq 2.$$

Piecewise Formula Combining the formulas for the two pieces of the graph, we obtain

$$f(x) = \begin{cases} x, & 0 \leq x < 1 \\ x - 1, & 1 \leq x \leq 2. \end{cases}$$

Now Try Exercise 43.

Absolute Value Function

The **absolute value function** $y = |x|$ is defined piecewise by the formula

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0. \end{cases}$$

The function is even, and its graph (Figure 1.17) is symmetric about the y -axis.

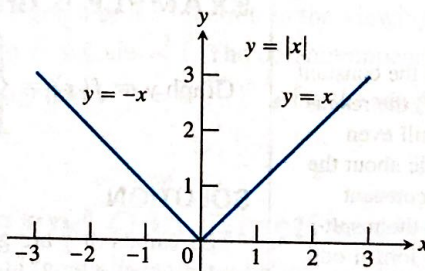
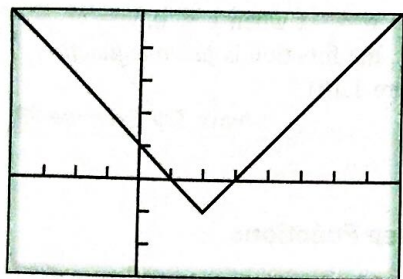


Figure 1.17 The absolute value function has domain $(-\infty, \infty)$ and range $[0, \infty)$.

$$y = |x - 2| - 1$$



$[-4, 8]$ by $[-3, 5]$

Figure 1.18 The lowest point of the graph of $f(x) = |x - 2| - 1$ is $(2, -1)$. (Example 7)

EXAMPLE 7 Using Transformations

Draw the graph of $f(x) = |x - 2| - 1$. Then find the domain and range.

SOLUTION

The graph of f is the graph of the absolute value function shifted 2 units horizontally to the right and 1 unit vertically downward (Figure 1.18). The domain of f is $(-\infty, \infty)$ and the range is $[-1, \infty)$.

Now Try Exercise 49.

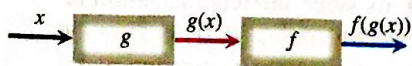


Figure 1.19 Two functions can be composed when a portion of the range of the first lies in the domain of the second.

Composite Functions

Suppose that some of the outputs of a function g can be used as inputs of a function f . We can then link g and f to form a new function whose inputs x are inputs of g and whose outputs are the numbers $f(g(x))$, as in Figure 1.19. We say that the function $f(g(x))$

(read “ f of g of x ”) is the **composite of g and f** . It is made by *composing* g and f in the order of first g , then f . The usual “stand-alone” notation for this composite is $f \circ g$, which is read as “ f of g .” Thus, the value of $f \circ g$ at x is $(f \circ g)(x) = f(g(x))$.

Composition Is Not Commutative

Function composition is *not* a commutative operation. In Example 8, $(f \circ g)(x) = x^2 - 7$, but $(g \circ f)(x) = (x - 7)^2$. For most functions, $f \circ g \neq g \circ f$.

EXAMPLE 8 Composing Functions

Find a formula for $f(g(x))$ if $g(x) = x^2$ and $f(x) = x - 7$. Then find $f(g(2))$.

SOLUTION

To find $f(g(x))$, we replace x in the formula $f(x) = x - 7$ by the expression given for $g(x)$.

$$f(x) = x - 7$$

$$f(g(x)) = g(x) - 7 = x^2 - 7$$

We then find the value of $f(g(2))$ by substituting 2 for x .

$$f(g(2)) = (2)^2 - 7 = -3$$

Now Try Exercise 51.

EXPLORATION 1 Composing Functions

Some graphers allow a function such as y_1 to be used as the independent variable of another function. With such a grapher, we can compose functions.

- Enter the functions $y_1 = f(x) = 4 - x^2$, $y_2 = g(x) = \sqrt{x}$, $y_3 = y_2(y_1(x))$, and $y_4 = y_1(y_2(x))$. Which of y_3 and y_4 corresponds to $f \circ g$? to $g \circ f$?
- Graph y_1 , y_2 , and y_3 and make conjectures about the domain and range of y_3 .
- Graph y_1 , y_2 , and y_4 and make conjectures about the domain and range of y_4 .
- Confirm your conjectures algebraically by finding formulas for y_3 and y_4 .

Quick Review 1.2 (For help, go to Appendix A1 and Section 1.2.)

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–6, solve for x .

1. $3x - 1 \leq 5x + 3$

2. $x(x - 2) > 0$

3. $|x - 3| \leq 4$

4. $|x - 2| \geq 5$

5. $x^2 < 16$

6. $9 - x^2 \geq 0$

In Exercises 7 and 8, describe how the graph of f can be transformed to the graph of g .

7. $f(x) = x^2$, $g(x) = (x + 2)^2 - 3$

8. $f(x) = |x|$, $g(x) = |x - 5| + 2$

In Exercises 9–12, find all real solutions to the equations.

9. $f(x) = x^2 - 5$

(a) $f(x) = 4$

(b) $f(x) = -6$

10. $f(x) = 1/x$

(a) $f(x) = -5$

(b) $f(x) = 0$

11. $f(x) = \sqrt{x + 7}$

(a) $f(x) = 4$

(b) $f(x) = 1$

12. $f(x) = \sqrt[3]{x - 1}$

(a) $f(x) = -2$

(b) $f(x) = 3$

Section 1.2 Exercises

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–4, (a) write a formula for the function and (b) use the formula to find the indicated value of the function.

- the area A of a circle as a function of its diameter d ; the area of a circle of diameter 4 in.
- the height h of an equilateral triangle as a function of its side length s ; the height of an equilateral triangle of side length 3 m
- the surface area S of a cube as a function of the length of the cube's edge e ; the surface area of a cube of edge length 5 ft
- the volume V of a sphere as a function of the sphere's radius r ; the volume of a sphere of radius 3 cm

In Exercises 5–12, (a) identify the domain and range and (b) sketch the graph of the function.

- | | |
|----------------------------------|------------------------------------|
| 5. $y = 4 - x^2$ | 6. $y = x^2 - 9$ |
| 7. $y = 2 + \sqrt{x - 1}$ | 8. $y = -\sqrt{-x}$ |
| 9. $y = \frac{1}{x - 2}$ | 10. $y = \sqrt[4]{-x}$ |
| 11. $y = 1 + \frac{1}{x}$ | 12. $y = 1 + \frac{1}{x^2}$ |

In Exercises 13–20, use a grapher to (a) identify the domain and range and (b) draw the graph of the function.

- | | |
|------------------------------------|---|
| 13. $y = \sqrt[3]{x}$ | 14. $y = 2\sqrt{3 - x}$ |
| 15. $y = \sqrt[3]{1 - x^2}$ | 16. $y = \sqrt{9 - x^2}$ |
| 17. $y = x^{2/5}$ | 18. $y = x^{3/2}$ |
| 19. $y = \sqrt[3]{x - 3}$ | 20. $y = \frac{1}{\sqrt{4 - x^2}}$ |

In Exercises 21–30, determine whether the function is even, odd, or neither.

- | | |
|--------------------------------------|------------------------------------|
| 21. $y = x^4$ | 22. $y = x + x^2$ |
| 23. $y = x + 2$ | 24. $y = x^2 - 3$ |
| 25. $y = \sqrt{x^2 + 2}$ | 26. $y = x + x^3$ |
| 27. $y = \frac{x^3}{x^2 - 1}$ | 28. $y = \sqrt[3]{2 - x}$ |
| 29. $y = \frac{1}{x - 1}$ | 30. $y = \frac{1}{x^2 - 1}$ |

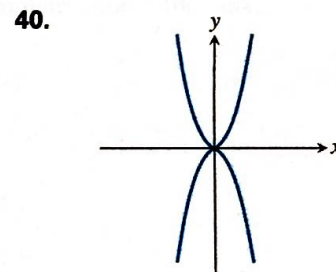
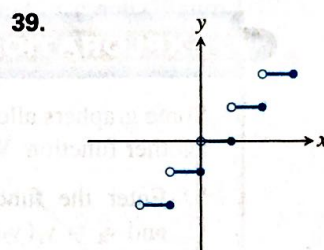
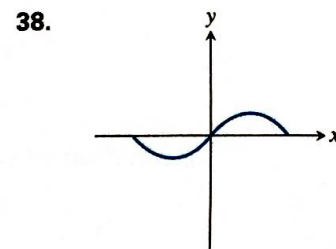
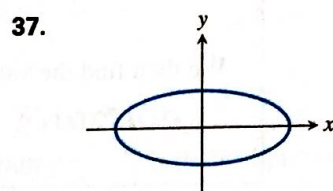
In Exercises 31–34, graph the piecewise-defined functions.

- 31.** $f(x) = \begin{cases} 3 - x, & x \leq 1 \\ 2x, & 1 < x \end{cases}$ **32.** $f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{x}, & x \geq 0 \end{cases}$
- 33.** $f(x) = \begin{cases} 4 - x^2, & x < 1 \\ (3/2)x + 3/2, & 1 \leq x \leq 3 \\ x + 3, & x > 3 \end{cases}$
- 34.** $f(x) = \begin{cases} x^2, & x < 0 \\ x^3, & 0 \leq x \leq 1 \\ 2x - 1, & x > 1 \end{cases}$

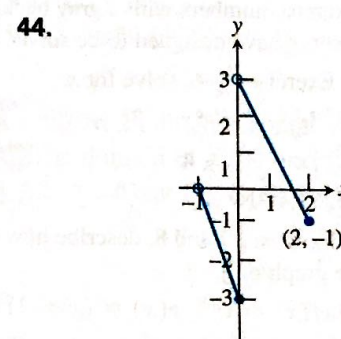
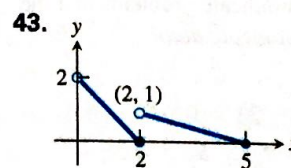
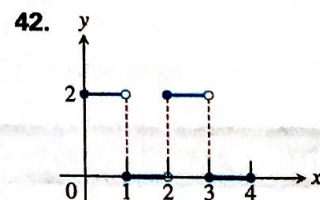
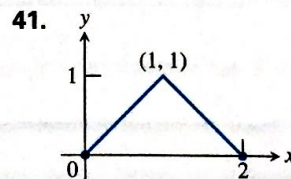
35. Writing to Learn The *vertical line test* to determine whether a curve is the graph of a function states: If every vertical line in the xy -plane intersects a given curve in at most one point, then the curve is the graph of a function. Explain why this is true.

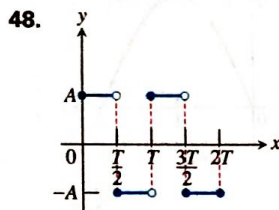
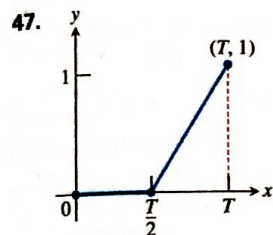
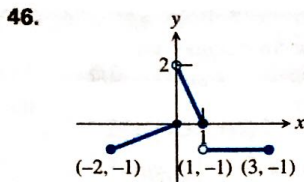
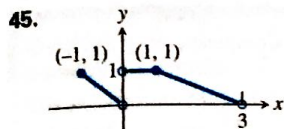
36. Writing to Learn For a curve to be *symmetric about the x -axis*, the point (x, y) must lie on the curve if and only if the point $(x, -y)$ lies on the curve. Explain why a curve that is symmetric about the x -axis is not the graph of a function, unless the function is $y = 0$.

In Exercises 37–40, use the vertical line test (see Exercise 35) to determine whether the curve is the graph of a function.



In Exercises 41–48, write a piecewise formula for the function.





In Exercises 49 and 50, (a) draw the graph of the function. Then find its (b) domain and (c) range.

49. $f(x) = -|3 - x| + 2$ 50. $f(x) = 2|x + 4| - 3$

In Exercises 51 and 52, find

- (a) $f(g(x))$ (b) $g(f(x))$ (c) $f(g(0))$
 (d) $g(f(0))$ (e) $g(g(-2))$ (f) $f(f(x))$

51. $f(x) = x + 5$, $g(x) = x^2 - 3$

52. $f(x) = x + 1$, $g(x) = x - 1$

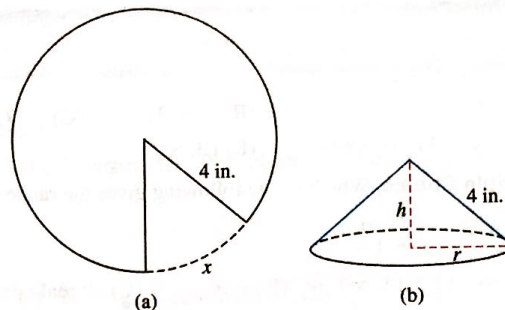
53. Copy and complete the following table.

	$g(x)$	$f(x)$	$(f \circ g)(x)$
(a)	?	$\sqrt{x - 5}$	$\sqrt{x^2 - 5}$
(b)	?	$1 + 1/x$	x
(c)	$1/x$?	x
(d)	\sqrt{x}	?	$ x , x \geq 0$

54. **The Cylindrical Can Problem** A cylindrical can is to be constructed so that its height h is equal to its diameter.

- (a) Write the volume of the cylinder as a function of its height h .
 (b) Write the total surface area of the cylindrical can (including the top and bottom) as a function of its height h .
 (c) Find the volume of the can if its total surface area is 54π square inches.

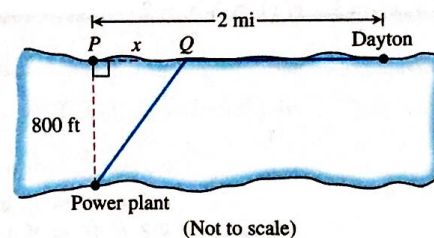
55. **The Cone Problem** Begin with a circular piece of paper with a 4-in. radius as shown in (a). Cut out a sector with an arc length of x . Join the two edges of the remaining portion to form a cone with radius r and height h , as shown in (b).



- (a) Explain why the circumference of the base of the cone is $8\pi - x$.
 (b) Express the radius r as a function of x .
 (c) Express the height h as a function of x .
 (d) Express the volume V of the cone as a function of x .

56. **Industrial Costs** Dayton Power and Light, Inc., has a power plant on the Miami River where the river is 800 ft wide. To lay a new cable from the plant to a location in the city 2 mi downstream on the opposite side costs \$180 per foot across the river and \$100 per foot along the land.

- (a) Suppose that the cable goes from the plant to a point Q on the opposite side that is x ft from the point P directly opposite the plant. Write a function $C(x)$ that gives the cost of laying the cable in terms of the distance x .
 (b) Generate a table of values to determine if the least expensive location for point Q is less than 2000 ft or greater than 2000 ft from point P .



Standardized Test Questions

57. **True or False** The function $f(x) = x^4 + x^2 + x$ is an even function. Justify your answer.
 58. **True or False** The function $f(x) = x^{-3}$ is an odd function. Justify your answer.

59. Multiple Choice Which of the following gives the domain of

$$f(x) = \frac{x}{\sqrt{9-x^2}}?$$

- (A) $x \neq \pm 3$ (B) $(-3, 3)$ (C) $[-3, 3]$
 (D) $(-\infty, -3) \cup (3, \infty)$ (E) $(3, \infty)$

60. Multiple Choice Which of the following gives the range of

$$f(x) = 1 + \frac{1}{x-1}?$$

- (A) $(-\infty, 1) \cup (1, \infty)$ (B) $x \neq 1$ (C) all real numbers
 (D) $(-\infty, 0) \cup (0, \infty)$ (E) $x \neq 0$

61. Multiple Choice If $f(x) = 2x - 1$ and $g(x) = x + 3$, which of the following gives $(f \circ g)(2)$?

- (A) 2 (B) 6 (C) 7 (D) 9 (E) 10

62. Multiple Choice The length L of a rectangle is twice as long as its width W . Which of the following gives the area A of the rectangle as a function of its width?

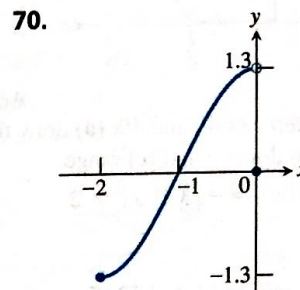
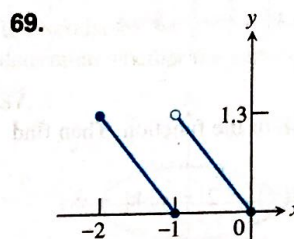
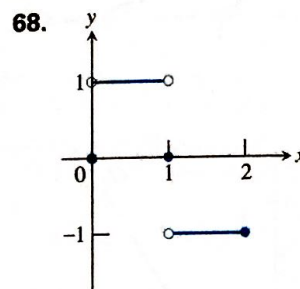
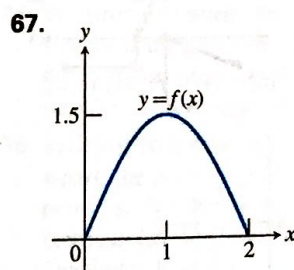
- (A) $A(W) = 3W$ (B) $A(W) = \frac{1}{2}W^2$
 (C) $A(W) = 2W^2$ (D) $A(W) = W^2 + 2W$
 (E) $A(W) = W^2 - 2W$

Explorations

In Exercises 63–66, (a) graph $f \circ g$ and $g \circ f$ and make a conjecture about the domain and range of each function. (b) Then confirm your conjectures by finding formulas for $f \circ g$ and $g \circ f$.

63. $f(x) = x - 7$, $g(x) = \sqrt{x}$
 64. $f(x) = 1 - x^2$, $g(x) = \sqrt{x}$
 65. $f(x) = x^2 - 3$, $g(x) = \sqrt{x + 2}$
 66. $f(x) = \frac{2x - 1}{x + 3}$, $g(x) = \frac{3x + 1}{2 - x}$

Group Activity In Exercises 67–70, a portion of the graph of a function defined on $[-2, 2]$ is shown. Complete each graph assuming that the graph is (a) even, (b) odd.



Extending the Ideas

71. Enter $y_1 = \sqrt{x}$, $y_2 = \sqrt{1-x}$ and $y_3 = y_1 + y_2$ on your grapher.
 (a) Graph y_3 in $[-3, 3]$ by $[-1, 3]$.
 (b) Compare the domain of the graph of y_3 with the domains of the graphs of y_1 and y_2 .
 (c) Replace y_3 by $y_1 - y_2$, $y_2 - y_1$, $y_1 \cdot y_2$, y_1/y_2 , and y_2/y_1 , in turn, and repeat the comparison of part (b).
 (d) Based on your observations in (b) and (c), what would you conjecture about the domains of sums, differences, products, and quotients of functions?
72. **Even and Odd Functions**
 (a) Must the product of two even functions always be even? Give reasons for your answer.
 (b) Can anything be said about the product of two odd functions? Give reasons for your answer.